Nonperturbative Corrections with Nonlocal Operators to Lifetime Ratios of Beauty Hadrons

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Abstract

The motion of spectator quarks in decay of a beauty hadron is a nonperturbative effect which can usually be neglected. We find that the motion in some decay channels, which contribute total decay widths of beauty hadrons, can not be neglected. The contributions from these decay channels to decay widths are proportional to certain averages of the squared inverse of the momentum carried by a spectator quark. This fact results in that these contributions, suppressed by $1/m_b^3$ formally, are effectively suppressed by $1/m_b$. We find these contributions can be factorized into products of perturbative coefficients and nonperturbative parameters. We calculate these coefficients and define these nonperturbative parameters in terms of HQET matrix elements. Since these parameters are unknown, we are unable to give numerical predictions in detail. But with a simple model it can be shown that these contributions can be large.

Although the heavy quark effective theory(HQET) is very successful to predict various properties of hadrons containing one heavy quark[1], but it is still difficult to predict the experimentally observed ratio of decay widths of B-meson and b-flavored baryon. The experimental results for lifetime ratios of beauty hadrons are[2]:

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.074 \pm 0.014, \qquad \frac{\tau(B_s)}{\tau(B_d)} = 0.948 \pm 0.038, \qquad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.796 \pm 0.052. \tag{1}$$

In HQET a systematic expansion in $1/m_b$ is employed to give theoretical predictions. At the leading order, the decay width of a beauty hadron equals the decay width of a free b-quark, hence the above ratio should be one, if higher-order correction is neglected. It is clearly that from Eq.(1) the ratio with Λ_b deviates from one substantially.

The correction to the leading-order result starts at order of m_b^{-2}. The decay width of a b-flavored hadron, denoted as H_b, takes in general the form

$$\Gamma(H_b) = \Gamma(b)\left(1 + \sum_{n=2} c_n \frac{\langle H_b|\mathcal{O}_n|H_b\rangle}{m_b^n}\right), \tag{2}$$

where c_n is a perturbative coefficient, $\mathcal{O}_n$'s are operators of HQET, whose matrix element represents nonperturbative effects in the decay. So far only contributions from local operators are considered. It should be noted that the correction at order of m_b^{-1} does not exist. It is expected that spectator effects will results in the difference between life times of b-flavored hadrons. Spectator effects appear at order of m_b^{-3}. These effects have been studied in [3, 4] at tree-level of QCD. Next-to-leading order corrections have been studied[5, 6, 7]. In these studies the nonperturbative effects at order of m_b^{-3} are parameterized with matrix elements of four quark operators. These matrix elements have been studied with lattice QCD[8] or with sum rule techniques(e.g., see [9]). With obtained values of matrix elements and effects at next-to-leading order the ratio becomes[7]:

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \qquad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \qquad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.90 \pm 0.05. \tag{3}$$

This prediction is closer to experiment, but there is still a discrepancy at order of 10% for Λ_b. Other attempts to explain the ratio in Eq.(1) can be found in [10]. It is interesting to note that the effects at order of m_b^{-3} studied before consist of local operators of four quark fields, in which two fields are fields of HQET for b-quark, while other two are for light quarks. These light quarks can be either as a part of H_b as a bound state, or they generate nonperturbatively soft decay products. Since the operators are local, the light quarks represented by the two quark fields carry zero momenta in the decay, i.e., the motion of spectator quarks is neglected.

The formula in Eq.(2) is based on a factorization in HQET, in which nonperturbative effects are factorized from perturbative effects. In general, one can expect that similar factorization formula can be obtained for inclusive productions of H_b, where nonperturbative effects related to H_b are parameterized by matrix elements of local operators, in corresponding to those matrix elements of $\langle H_b|\mathcal{O}_n|H_b\rangle$. At the leading order of m_b^{-1} the production rate of H_b can be factorized as a product of the production rate of a free b-quark with a matrix element of HQET. The matrix element can be interpreted as the probability for the transition of the b-quark into H_b. Inclusive productions based on such a factorization have been studied, predictions at leading order of m_b^{-1} have been made for production at e^+e^- colliders and for polarization of heavy vector meson[11, 12]. In these cases a good agreement with experiment were found.

Recently, such a factorization was employed to explain the asymmetry between production rates of D^+ and of D^- in their inclusive productions, and also asymmetries for other heavy flavored hadrons[13, 14, 15, 16]. In these works contributions from quark recombination to production rates are studied, in which a heavy flavored hadron like H_b is produced by combining a b-quark with a light antiquark $\bar{q}$. Including these contributions the asymmetry can be explained[13, 14, 15]. The nonperturbative effect of the recombination can be represented as matrix elements of four quark fields in the production rate. But it is found that the momentum of $\bar{q}$ can not be taken as zero, because it will generate a type of infrared singularities in the production amplitude of $b\bar{q}$. This type of singularities can be regularized by noting the fact that the light antiquark inside H_b carries a small momentum at order of Λ_{QCD} and its effect is nonperturbative. One needs new matrix elements beside these matrix elements of *local* operators, corresponding to those in Eq.(2), to incorporate this nonperturbative effect. These new matrix elements are found to be *nonlocal* matrix elements, whose definitions are given in [16]. It turns out that the contributions to the production rates due to quark recombination is proportional some averages of the inverse of momenta carried by the light antiquark. This results in that the contributions are significant and the large asymmetry observed in experiment can be explained in this way. The effect in quark recombination in production c-flavored baryon[17] and in production b-flavored jet in Z^0-decay[18] has also been studied.

In this letter we point out similar contributions also exist as corrections to lifetime ratios. These contributions formally are suppressed by m_b^{-3}, but they are proportional to the square of the inverse of the momentum carried by a light antiquark in H_b or by a light antiquark which generates soft decay products nonperturbatively. Therefore these corrections are only suppressed by m_b^{-1} effectively. Similar situation also appears in the decay $B \to \gamma\ell\nu$, where the decay amplitude is proportional to a certain average of the inverse of the momentum carried by the light quark inside the B meson. This decay mode has been studied in detail in [19, 20]. In this letter we will study this type of contributions to lifetimes of beauty hadrons.

The effective weak Hamiltonian for the decay of H_b at the scale $\mu = m_b$ is:

$$\begin{aligned} H_{eff} &= \frac{G_F}{\sqrt{2}} V_{cb} \sum_{q=d,s} \Big\{ c_1(m_b)[V_{uq}^* \bar{q}_L \gamma^\mu u_L \bar{c}_L \gamma_\mu b_L + V_{cq}^* \bar{q}_L \gamma^\mu c_L \bar{c}_L \gamma_\mu b_L] \\ &\quad + c_2(m_b)[V_{uq}^* \bar{c}_L \gamma^\mu u_L \bar{q}_L \gamma_\mu b_L + V_{cq}^* \bar{c}_L \gamma^\mu c_L \bar{q}_L \gamma_\mu b_L] \Big\}, \end{aligned} \tag{4}$$

where we neglected the suppressed transition $b \to u$ and $q_L = \frac{1}{2}(1-\gamma_5)q$. c_1 and c_2 are Wilson coefficients. The above mentioned contributions come from these decays at the tree-level:

$$\begin{aligned} H_b(P) &\to c(p_1) + q(p_2) + G(k) + X, \\ H_b(P) &\to c(p_1) + \bar{u}(p_2) + G(k) + X, \\ H_b(P) &\to c(p_1) + \bar{c}(p_2) + G(k) + X. \end{aligned} \tag{5}$$

where momenta are given in brackets. We will not consider a c-quark as a spectator, hence the decay into a lepton pair with a gluon and other unobserved states X will not lead to contributions we consider. In general the gluon can have any possible momentum. If the momentum is large, one can use perturbative QCD. If the momentum is small, the contributions can have an infrared singularity, reflecting the fact that the gluon can not be taken as a perturbative gluon. However, it turns out that the contributions we are interesting in are free of infrared singularities. This will be discussed in detail. We will give some detail for calculation the contribution from the process $H_b \to c + q + G + X$.

With the effective Hamiltonian the decay amplitude can be written as:

$$\mathcal{T} = \int \frac{d^4 q_1}{(2\pi)^4} A_{ij}(q_1, q_2) \int d^4 x_1 e^{iq_1\cdot x_1} \langle X | u_i(x_1) b_j(0) | H_b \rangle, \tag{6}$$

where i, j stand for spin- and color indices. A_{ij} is the scattering amplitude for $u(q_1) + b(q_2) \to c(p_1) + q(p_2) + G(k)$ in which the quarks in the initial state are off-shell in general and $q_1 + q_2 = p_1 + p_2 + k$. With the translational symmetry the contribution to the decay width can be written:

$$\begin{aligned} \delta\Gamma_{cqg} &= \int d\Gamma_{cqg} \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} A_{ij}(q_1, q_2) \left(\gamma^0 A^\dagger(q_3, q_4)\gamma^0\right)_{lk} \\ &\quad \cdot(-1) \int d^4x_1 d^4x_3 d^4x_4 \; e^{iq_1\cdot x_1 - iq_3\cdot x_3 - ix_4\cdot q_4} \langle H_b | \bar{b}_l(x_4) u_i(x_1) \bar{u}_k(x_3) b_j(0) | H_b \rangle, \end{aligned} \tag{7}$$

with $q_4 = p_1 + p_2 + k - q_3$. In the above we exchanged the order of u-quark fields and this gives the $-$ sign in the second line. The integration measure for the phase space of three particles cqg is denoted as $d\Gamma_{cqg}$. We use nonrelativistic normalization for the state H_b and b-quark. The above contribution can be illustrated by Fig.1., where one of 16 diagrams is shown explicitly.

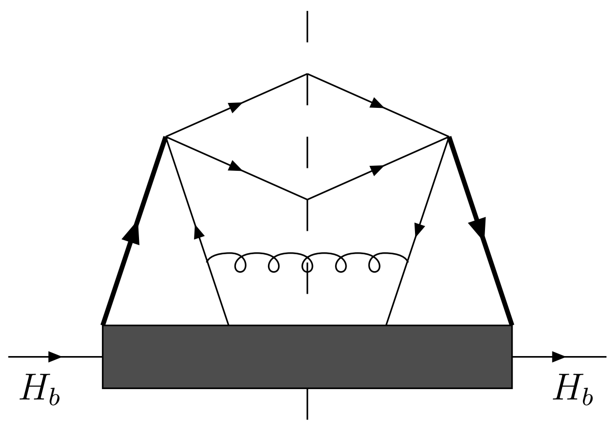

Fig.1

Figure 1: Diagrams for the contributions to the decay width. Other diagrams are obtained by changing attachments of the gluon. The thick line is for the b-quark. The broken line is the cut, the black box represents nonperturbative effect in the decay, its expression is given in Eq.(9).

For b-quark field $b(x)$ one can use the m_b^{-1} expansion:

$$b(x) = e^{-im_b v\cdot x}\left(h(x) + \cdots\right), \quad \bar{b}(x) = e^{+im_b v\cdot x}\left(\bar{h}(x) + \cdots\right), \tag{8}$$

where v the four velocity of H_b. The $\cdots$ represent higher orders in m_b^{-1}, which can be neglected in this letter. With v any vector B^μ can be decomposed as $B^\mu = v \cdot B v^\mu + B_\perp^\mu$ with $B_\perp \cdot v = 0$. Taking the leading term the Fourier transformed matrix element becomes

$$\begin{aligned} &\int d^4x_1 d^4x_3 d^4x_4 \; e^{iq_1\cdot x_1 - iq_3\cdot x_3 - ix_4\cdot q_4} \langle H_b | \bar{b}_l(x_4) u_i(x_1) \bar{u}_k(x_3) b_j(0) | H_b \rangle \\ &= \int d^4x_1 d^4x_3 d^4x_4 \; e^{iq_1\cdot x_1 - iq_3\cdot x_3 - ix_4\cdot(q_4 - m_b v)} \langle H_b | \bar{h}_l(x_4) u_i(x_1) \bar{u}_k(x_3) h_j(0) | H_b \rangle + \cdots. \end{aligned} \tag{9}$$

Since we extract the large momentum $m_b v$ by using the expansion in Eq.(8), the space-time dependence of the matrix element in the second line in Eq.(10) is controlled by the soft scale Λ_{QCD}. The x-dependence of h fields can be safely neglected. If we can neglect the x-dependence of the light quark fields, then the contribution will proportional to matrix elements of *local* four-quark operators, which appear at order of m_b^{-3} in Eq.(2). This implies that the light quark $\bar{q}$ will carry zero momentum. But the amplitude A with the zero momentum is divergent or proportional to the inverse of the light quark mass m_u if we do not neglect this mass. This divergence represents some new nonperturbative effects which can not be represented by local four quark matrix elements. To study the divergence, we introduce a light-cone coordinate system in which the two light-cone vectors are l and n respectively and $l \cdot n = 1$. In the light-cone coordinate system the emitted gluon has the momentum $k^\mu = \sqrt{2} k^0 l^\mu$. The expansion in q_1 reads:

$$A_{ij}(q_1, m_b v) = -\frac{m_b}{q_1 \cdot l} T_{ij} + \cdots, \tag{10}$$

where $\cdots$ stand for higher order terms of q_1 and we set $q_2 = m_b v$. T_{ij} reads:

$$\begin{aligned} T_{ij} &= i\frac{g_s G_F}{\sqrt{2} m_b} V_{cb} V_{uq}^* \Big\{ c_1 [\bar{u}(p_1)\gamma^\mu(1-\gamma_5)]_j [\bar{u}(p_2) T^a \gamma_\mu (1-\gamma_5)\gamma\cdot\varepsilon^*(k)\gamma\cdot l]_i \\ &\qquad - c_2 [\bar{u}(p_2)\gamma^\mu(1-\gamma_5)]_j [\bar{u}(p_1) T^a \gamma_\mu (1-\gamma_5)\gamma\cdot\varepsilon^*(k)\gamma\cdot l]_i \Big\} \\ &= [T'\gamma\cdot l]_{ji} \end{aligned} \tag{11}$$

where $\varepsilon^*(k)$ is the polarization vector of the gluon and we used $k\cdot\varepsilon^*(k) = 0$. T_{ij} only gets contribution from Fig.1., it does not depend on q_1. It should be noted that the matrix T can always be written in the form as in the last line of Eq.(10) for our processes in Eq.(5). Keep only the leading term for A_{ij}, some integrations in Eq.(7) can be performed. We obtain:

$$\begin{aligned} \delta\Gamma_{cqg} &= \int d\Gamma_{cqg} \int \frac{d\eta_1}{2\pi}\frac{d\eta_3}{2\pi}(2\pi)^4\delta^4(m_b v - p_1 - p_2 - k)\frac{1}{2\eta_1\eta_3} T'_{ji}\left(\gamma^0 T'^\dagger \gamma^0\right)_{kl} \\ &\quad \cdot(-m_b^2)\int d\omega_1 d\omega_3\ e^{i\eta_1\omega_1 m_b - i\eta_3\omega_3 m_b}\langle H_b|\bar{h}_l(0)[\gamma^- u]_i(\omega_1 l)[\bar{u}\gamma^-]_k(\omega_3 l) h_j(0)|H_b\rangle. \end{aligned} \tag{12}$$

where we have used $q_i \cdot l = \eta_i m_b$ for $i = 1, 3$ and moved $\gamma^- = \gamma\cdot l$. into the matrix element. It is interesting to note that the T_{ij} is finite when the gluon carries null momentum. This indicates that the decay width will be free from infrared singularities when we perform the phase-space integration. If one keeps the next-to-leading order in q_1 or q_3, infrared singularities will appear, but these singularities may be cancelled by some virtual corrections partly and absorbed into four quark matrix elements. The contribution at this order will be suppressed by $(M_{H_b} - m_b)/m_b$ in comparison with that from the leading order. In our approximation the decay width will be proportional to the integral of the Fourier transformed matrix element. The Fourier transformed matrix element can be parameterized as:

$$\begin{aligned} &\frac{1}{m_b}\int\frac{d\eta_1}{\eta_1}\frac{d\eta_3}{\eta_3}\int\frac{d\omega_1}{2\pi}\frac{d\omega_3}{2\pi}\ e^{i\eta_1\omega_1 m_b - i\eta_3\omega_3 m_b}\langle H_b|\bar{h}_l(0)[\gamma^- u]_i(\omega_1 l)[\bar{u}\gamma^-]_k(\omega_3 l) h_j(0)|H_b\rangle \\ &= \frac{1}{3}[(P_v)_{jk}(P_v)_{il}\,\mathcal{S}_{H_b}^{(u,1)} - (\gamma_5 P_v)_{jk}(P_v\gamma_5)_{il}\,\mathcal{P}_{H_b}^{(u,1)} - (\gamma_T^\mu P_v)_{jk}(P_v\gamma_{T\mu})_{il}\,\mathcal{V}_{H_b}^{(u,1)} \\ &- (\gamma_T^\mu\gamma_5 P_v)_{jk}(P_v\gamma_{T\mu}\gamma_5)_{il}\,\mathcal{A}_{H_b}^{(u,1)}] + \frac{1}{2}[(P_v T^a)_{jk}(P_v T^a)_{il}\,\mathcal{S}_{H_b}^{(u,8)} - (\gamma_5 P_v T^a)_{jk}(P_v\gamma_5 T^a)_{il}\,\mathcal{P}_{H_b}^{(u,8)} \\ &- (\gamma_T^\mu P_v T^a)_{jk}(P_v\gamma_{T\mu}T^a)_{il}\,\mathcal{V}_{H_b}^{(u,8)} - (\gamma_T^\mu\gamma_5 T^a P_v)_{jk}(P_v\gamma_{T\mu}\gamma_5 T^a)_{il}\,\mathcal{A}_{H_b}^{(u,8)}], \end{aligned} \tag{13}$$

with

$$\gamma_T^\mu = \gamma^\mu - v\cdot\gamma v^\mu, \qquad P_v = \frac{1+\gamma\cdot v}{2} \tag{14}$$

The above matrix elements are defined in the rest frame of H_b. The eight coefficients $\mathcal{S}_{H_b}^{(u,1)}, \cdots$ in Eq.(13) are dimensionless, their values depend on H_b. These coefficients multiplied with the factor $\sqrt{2v^0}$ are Lorentz covariant because we take nonrelativistic normalization for the state. We work in the light-cone gauge $l\cdot G = 0$. In other gauges gauge links should be added into the above matrix element between quark fields to make it gauge invariant. If we replace in the integral the factor $(\eta_1\eta_3)^{-1}$ with a constant, the eight coefficients will be at order of Λ_{QCD}^3/m_b^3. Taking this fact into account, the coefficients are at order of $m_b^2/(M_{H_b}-m_b)^2\cdot\Lambda_{QCD}^3/m_b^3 \sim \Lambda_{QCD}/m_b$. Therefore, the contribution to the total decay width is effectively suppressed by Λ_{QCD}/m_b. It is now straightforward to obtain:

$$\begin{aligned}
\delta\Gamma_{cqg} &= -64 g_s^2 G_F^2 |V_{cb}V_{uq}^*|^2 m_b \cdot \int d\Gamma_{cqg}(2\pi)^4\delta^4(m_b v - p_1 - p_2 - k)p_1\cdot p_2 \\
&\quad \cdot\Big\{(c_1^2+c_2^2)\left[\frac{4}{3}\left(f_1^{u/H_b}+f_2^{u/H_b}\right)+\frac{16}{6}\left(f_3^{u/H_b}+f_4^{u/H_b}\right)\right] \\
&\quad +2c_1c_2\left(f_3^{u/H_b}+f_4^{u/H_b}\right)\Big\}.
\end{aligned} \tag{15}$$

The parameters $\mathcal{W}_{S-P}^{(u,i)}, \cdots$ are defined as:

$$\begin{aligned}
f_1^{u/H_b} &= \mathcal{S}_{H_b}^{(u,1)} + \mathcal{P}_{H_b}^{(u,1)}, \qquad f_2^{u/H_b} = \mathcal{V}_{H_b}^{(u,1)} + \mathcal{A}_{H_b}^{(u,1)}, \\
f_3^{u/H_b} &= \mathcal{S}_{H_b}^{(u,8)} + \mathcal{P}_{H_b}^{(u,8)}, \qquad f_4^{u/H_b} = \mathcal{V}_{H_b}^{(u,8)} + \mathcal{A}_{H_b}^{(u,8)},
\end{aligned} \tag{16}$$

Performing the phase-space integration we obtain:

$$\begin{aligned}
\delta\Gamma_{cqg} &= -32\pi\alpha_s\Gamma_0|V_{uq}^*|^2\Big\{1-6z+3z^2+2z^3-6z^2\ln(z)\Big\} \\
&\quad \cdot\Big\{\frac{4}{3}(c_1^2+c_2^2)[f_1^{u/H_b}+f_2^{u/H_b}]+[\frac{8}{3}(c_1^2+c_2^2)+2c_1c_2)[f_3^{u/H_b}+f_3^{u/H_b}]\Big\},
\end{aligned} \tag{17}$$

where $z=\frac{m_c^2}{m_b^2}$ and

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2. \tag{18}$$

Similarly, one can work out the contributions of other two processes. To present our results we introduce:

$$\begin{aligned}
F_{cug} &= \frac{1}{3}[2-9z+18z^2-11z^3+6z^3\ln(z)], \\
\tilde{F}_{cug} &= \frac{1}{6}[11-54z+36z^2-2z^3+9z^4-12z^2(3+z)\ln(z)], \\
F_{ccg} &= \frac{2\beta}{3}\Big[1-7z+6z^2\Big]-2(5+2z^3)\ln\Big[1-3z-\beta(1-z)\Big] \\
&\quad +4z^3\ln[z(1+\beta)]+10\ln\Big[\frac{4z^2(1-\beta)}{(1+\beta)^2}\Big], \\
\tilde{F}_{ccg} &= \frac{\beta}{6}\Big[11-86z-6z^2+108z^3\Big]-10\ln\Big[\frac{4z^2(1-\beta)}{(1+\beta)^2}\Big]+2\Big(5-3z^2+2z^3 \\
&\quad -9z^4\Big)\ln\Big[1-3z-\beta(1-z)\Big]+2z^2\Big(3-2z+9z^2\Big)\ln\Big[z(1+\beta)\Big],
\end{aligned} \tag{19}$$

with $\beta=\sqrt{1-4z}$. The processes $H_b(P)\to c(p_1)+\bar{u}(p_2)+G(k)+X$ gives the contribution:

$$\begin{aligned}\delta\Gamma_{cug} &= 32\pi\alpha_s\Gamma_0|V_{uq}^*|^2\Big\{\frac{2}{3}c_1^2\Big[f_1^{q/H_b}\,F_{cug}+f_2^{q/H_b}\tilde{F}_{cug}\Big]\\&\quad+\Big[\frac{4}{3}c_1^2+\frac{1}{2}\Big(c_2^2+2c_1c_2\Big)\Big]\Big[f_3^{q/H_b}\,F_{cug}+f_4^{q/H_b}\tilde{F}_{cug}\Big]\Big\},\end{aligned}\tag{20}$$

Replacing the $\bar{u}$ quark with a $\bar{c}$ quark, we obtain the contribution $\delta\Gamma_{c\bar{c}g}$ from the process $H_b(P)\to c(p_1)+\bar{c}(p_2)+G(k)+X$.

We use the parameters:

$$m_b=4.8\mathrm{GeV},\qquad z=0.085.\tag{21}$$

Correspondingly we have $\alpha_s=0.18,\ c_1=1.105$ and $c_2=-0.245$. The lifetime ratio can be predicted by using experimental values as:

$$\frac{\tau(H_b)}{\tau(B_d)}-1=\frac{\Gamma(\bar{B}^0)}{\Gamma(H_b)}-1=\left(\frac{\Gamma(\bar{B}^0)}{\Gamma(H_b)}\right)_{exp}\cdot\frac{\Gamma_0}{\Gamma_{exp}(\bar{B}^0)}\cdot\left(\frac{\Gamma(\bar{B}^0)-\Gamma(H_b)}{\Gamma_0}\right)_{theory}.\tag{22}$$

For our numerical estimation we only take effects of valence quarks into account and use isospin symmetry. We obtain:

$$\begin{aligned}\frac{\tau(B^-)}{\tau(B^0)} &= 1+5.06f_1^{u/B^-}+7.82f_2^{u/B^-}+8.55f_3^{u/B^-}+13.26f_4^{u/B^-},\\ \frac{\tau(B_s)}{\tau(B^0)} &= 1-0.94f_1^{s/B_s}+1.50f_1^{u/B^-}-2.55f_2^{s/B_s}+4.07f_2^{u/B^-}-1.59f_3^{s/B_s}\\&\quad+2.56f_3^{u/B^-}-4.34f_4^{s/B_s}+6.94f_4^{u/B^-},\\ \frac{\tau(\Lambda_b)}{\tau(B^0)} &= 1+1.36f_1^{d/\Lambda_b}+1.20f_1^{u/B^-}-0.69f_2^{d/\Lambda_b}+3.25f_2^{u/B^-}+2.27f_3^{d/\Lambda_b}\\&\quad+2.04f_3^{u/B^-}-1.23f_4^{d/\Lambda_b}+5.53f_4^{u/B^-}.\end{aligned}\tag{23}$$

At moment no information for these nonperturbative parameters is available. This prevents us to give numerical predictions. If we use the approximation of vacuum saturation, one may estimate the correction to lifetimes of beauty mesons. We use this approximation below to give some numerical predictions for mesons.

If one uses the vacuum saturation approximation, the nonperturbative parameters are either zero or can be expressed with the light-cone wave function ϕ_+ of B-meson, which has been studied extensively(See e.g., [21, 22, 23, 24]. The light cone wave function is defined in the light-cone gauge as[24]:

$$\langle 0|\bar{q}(zl)\gamma\cdot l\Gamma h(0)|\bar{B}(v)\rangle=-\frac{i}{2}F_B\mathrm{Tr}\left[\gamma_5\gamma\cdot l\Gamma P_v\right]\phi_+(z),\tag{24}$$

where F_B is the decay constant in HQET. At tree-level it is related to the decay constant f_B of full QCD via $F_B=f_B\sqrt{M_B}$. Γ is an arbitrary Dirac matrix. Under the approximation of vacuum saturation, one can show with Eq.(24) that all nonperturbative parameters in Eq.(13) are zero except $\mathcal{P}_B^{(q,1)}$. Hence only $f_1^{q/B}$ is not zero and it is given by:

$$f_1^{q/B}=\frac{f_B^2M_B}{4m_b}\cdot\frac{1}{\lambda_B^2},\qquad\frac{1}{\lambda_B}=\left|\int\frac{dk}{k}\phi_+(k)\right|,\tag{25}$$

where $\phi_+(k)$ is the Fourier transformed function of $\phi_+(z)$. The parameter λ_B is at order of λ_{QCD}. It is estimated to be in the range $0.35 \sim 0.6$MeV[20, 24, 25, 26]. With this we obtain the following numerical results in the approximation of vacuum saturation:

$$\begin{aligned} \frac{\tau(B^-)}{\tau(B^0)} &= 1 + (0.15 \sim 0.45), \\ \frac{\tau(B_s)}{\tau(B^0)} &= 1 + (0.017 \sim 0.049) \end{aligned} \tag{26}$$

where we have used $SU(3)$ isospin symmetry. Comparing experimental results, the predicted ratio of B^- is too large. It should be noted that the approximation of vacuum saturation is not a well-established approximation, it serves only for a rough estimation. A detailed study of these nonperturbative parameters is required. If one assumes that the nonperturbative parameters of Λ_b is at the same order of $f_1^{q/B}$ determined as above, one can also have a large correction to the ratio of Λ_b.

To summarize: In this letter we have studied corrections of spectator quarks to lifetime ratios of beauty hadrons. Formally, these corrections are at order of m_b^{-3}. We find some decay channels which lead to that these corrections are proportional to certain averages of the squared inverse of the momentum carried by a spectator quark. Hence these corrections are effectively at order of m_b^{-1}. The corrections are calculated at tree-level and nonperturbative effects are parameterized with nonlocal operators. Since the nonperturbative parameters are unknown, we are unable to give numerical predictions in detail. With a simple model one can estimate these parameters and the obtained ratio of B^- is too large in comparison with experiment. However, estimations of these parameters in order of magnitude indicate that these corrections may be large enough to accommodate experimental results. A detail study of these nonperturbative parameters is required to have detailed predictions.

Acknowledgements

This work of is supported by National Nature Science Foundation of P. R. China.

References

[1] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113, *ibid.* B237 (1990) 527,
E. Eichten and B. Hill, Phys. Lett. B234 (1990) 511,
B. Grinstein, Nucl. Phys. B339 (1990) 253,
H. Georgi, Phys. Lett. B240 (1990) 447.

[2] E. Barberio *et al.*, Prepared for Workshop on CKM Unitarity Triangle (CERN 2002-2003), Geneva, Switzerland, 13-16 Feb 2002, published in "Geneva 2002, The CKM matrix and the unitarity triangle" 39-141, http://ckm-workshop.web.cern.cn/.

[3] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B293 (1992) 430, Erratum-*ibid.* B297 (1992) 477

[4] M. Neubert and C.T. Sachajda, Nucl. Phys. B483 (1997) 339

[5] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl.Phys.B639 (2002) 389

[6] M. Ciuchini, E. Franco, V. Lubicz and F. Mescia, Nucl.Phys.B625 (2002) 211

[7] E. Franco, V. Lubicz, F. Mescia and C. Tarantino, Nucl. Phys. B633 (2002) 212

[8] M. Di Pierro and C.T. Sachrajda [UKQCD Collaboration], Nucl. Phys. B534 (1998) 373,
M. Di Pierro, C. T. Sachrajda and C. Michael [UKQCD collaboration], Phys. Lett. B468 (1999) 143,
M. Di Pierro and C.T. Sachrajda [UKQCD collaboration], Nucl. Phys. Proc. Suppl. **73** (1999) 384,
D.Becirevic, Talk given at International Europhysics Conference on High-Energy Physics (HEP 2001), Budapest, Hungary, 12-18 Jul 2001. Published in "Budapest 2001, High energy physics" hep2001/098, arXiv: hep-ph/0110124.

[9] P. Colangelo and F. De Fazio, Phys. Lett. B387 (1996) 371,
M. S. Baek, J. Lee, C. Liu and H. S. Song, Phys. Rev. D57 (1998) 4091,
H. Y. Cheng and K. C. Yang, Phys. Rev. D59 (1999) 014011,
C. S. Huang, C. Liu and S. L. Zhu, Phys. Rev. D61 (2000) 054004,

[10] G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, Phys.Lett. B382 (1996) 409,
Y.A Yan, Y.L Wu and W.Y Wang, Int.J.Mod.Phys. A15 (2000) 2735

[11] J.P. Ma, Nucl.Phys. B506 (1997) 329

[12] J.P. Ma, Nucl.Phys. B622 (2002) 416

[13] E. Braaten, Y. Jia and T. Mehen, Phys. Rev. Lett. **89** (2002) 122002

[14] E. Braaten, Y. Jia and T. Mehen, Phys. Rev. D66 (2002) 014003

[15] E. Braaten, Y. Jia and T. Mehen, Phys. Rev. D66 (2002) 034003

[16] C.-H. Chang, J.P. Ma and Z.-G. Si, Phys. Rev. D68 (2003) 014018

[17] E. Braaten, M. Kusunoki, Y. Jia and T. Mehen, hep-ph/0304280

[18] Y. Jia, hep-ph/0305172

[19] E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B649 (2003) 349,
G.P. Korchemsky, D. Pirjol and T.M. Yan, Phys. Rev. D61 (2000) 114510,
S. Descotes-Genon and C.T. Sachrajda, Phys. Lett. B557 (2003) 213,
S.W. Bosch, R.J. Hill, B.O. Lange and M. Neubert, Phys. Rev. D67 (2003) 094014.

[20] S. Descotes-Genon and C.T. Sachrajda, Nucl. Phys. B650 (2003) 356

[21] A. Szczepaniak, E.M. Henley and S.J. Brodsky, Phys. Lett. B243 (1990) 287

[22] A.G. Grozin and M. Neubert, Phys. Rev. D55 (1977) 272

[23] H. Kawamura, J. Kodaira, C.F. Qiao and Y. Tanaka, Phys. Lett. B523 (2001) 111, Erratum-*ibid.*, B536 (2002) 344

[24] V.M. Braun, D.Yu. Ivanov and G.P. Korchemsky, arXive: hep-ph/0309330.

[25] P. Ball and E. Kou, JHEP **0304** (2003) 029

[26] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. **83** (1999) 1914, Nucl. Phys. B591 (2000) 313